Unit IV More Applications of Kinematics and Dynamics

Things to Remember

Period: The time it takes to go through one revolution or cycle. Measured in seconds.

$$T = \frac{t}{cycles}$$

Frequency: How often something repeats itself; the number of cycles per second. Measured in Hz (/s).

$$f = \frac{cycles}{t}$$

Do remember that T and f are inverse of each other:

$$T = \frac{1}{f} \; and \; f = \frac{1}{T}$$

Ex. A conventional, non-SSD hard drive of a computer is a metal disc that spins while data is written or read from it. The specs listed for a computer you are looking at is that the hard drive spins at 7200 rpm.

a) What does this mean?

- rotates 7200 every minute

b) Find its frequency and explain this measurement.

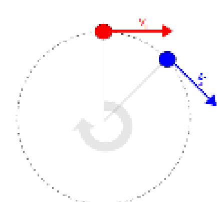
$$f = \frac{cycles}{t} = \frac{7200}{60s} = 120\,Hz$$

c) Find its period.

$$T = \frac{1}{f} = \frac{1}{120Hz} = 0.0083\,s$$

Circular Motion

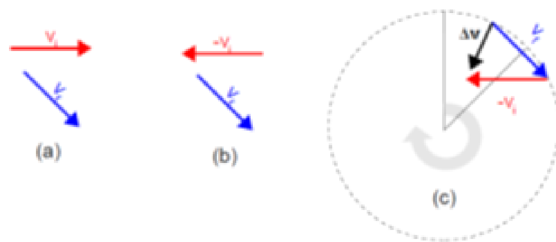
Think of spinning a yo-yo over your head. How would you describe its motion? Clearly, it moves in the path of a circle.



For all of our questions we do, assume that the object travelling in the circle is moving at a constant speed. Do not confuse speed with velocity, as the object is still accelerating due to a change in direction as it moves around the circle.

The velocity vector points along a tangent to the circle, in the direction that the object would tend to move if it were suddenly released.

The acceleration acts in the same direction as the change in velocity. The picture below shows the original vectors from the picture above (a) changed to show $-v_i$ (b) in order to find Δv (c). Notice the direction of Δv will always point towards the centre of the circle.



While travelling in the path of a circle, the acceleration is called centripetal acceleration. It always acts inward, toward the centre of the circle, in the same direction as the change in velocity, perpendicular to the velocity vector. Centripetal comes from Greek meaning "centre-seeking".

Since we are moving at a constant speed, we can modify $\vec{v} = \frac{\Delta d}{\Delta t}$ to get:

$$\vec{v} = \frac{2\pi r}{t}$$

$\vec{v} = \frac{2\pi r}{t}$	Where v = velocity r = radius of circle t = time for one revolution

Since t would be the time to go around one complete circle, you could substitute in period:

$\vec{v} = \frac{2\pi r}{T}$	Where v = velocity r = radius of circle T = Period

Centripetal Acceleration

The magnitude of the centripetal acceleration is given by:

$\vec{a} = \frac{\vec{v}^2}{r}$	Where a = acceleration towards centre of circle r = radius of circle v = velocity of object
$\vec{a} = \frac{4\pi^2 r}{T^2}$	T = period of revolution
$\vec{a} = 4\pi^2 r f^2$	f = frequency of revolution

Ex. A DVD disc has a diameter of 12.0 cm and a rotational period of 0.100 s. Determine the centripetal acceleration at the outer edge of the disc

$d = 12.0\text{ cm} = 0.120\text{ m}$
$r = 0.0600\text{ m}$
$T = 0.100\text{ s}$
$\vec{a}_c = ?$

$a_c = \frac{4\pi^2 r}{T^2}$

$\vec{a}_c = \frac{4\pi^2(0.0600\text{ m})}{(0.100\text{ s})^2}$

$\boxed{\vec{a}_c \approx 237\text{ m/s}^2 \text{ [towards centre]}}$

Centripetal Force

Since we have acceleration when going around a circle, we must also have an accompanying force. Using Newton's Second Law, we can derive a formula for this force that is in the same direction as the acceleration:

$\vec{F}_c = m\vec{a}_c$	Where F_c = centripetal force m = mass of object a_c = centripetal acceleration

Furthermore, we can substitute any of our formulas for centripetal acceleration into Newton's Second law to get the following:

$\vec{F}_c = m\frac{\vec{v}^2}{r}$	$\vec{F}_c = \frac{4\pi^2 mr}{T^2}$	$\vec{F}_c = 4\pi^2 mrf^2$

Ex. Determine the magnitude of the centripetal force exerted by the rim of a dragster's wheel on a 45.0-kg tire. The tire has a 0.480-m radius and is rotating at a speed of 30.0 m/s

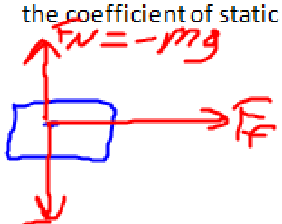
$F_c = ?$
$m = 45.0kg$
$r = 0.480m$
$v = 30.0m/s$

$$\vec{F}_c = m\frac{v^2}{r}$$

$$F_c = 45.0kg\left(\frac{(30.0m/s)^2}{0.480m}\right)$$

$$\boxed{F_c = 84\,400N}$$

Ex. Determine the maximum speed with which a 1500 kg car can safely travel around a circular track of 80.0 m if the coefficient of static friction between the tire and road is 0.30.



$F_g = mg$
$m = 1500kg$
$r = 80.0m$
$\mu_s = 0.30$
$v = ?$

$$F_f = F_c$$

$$\mu_s F_N = m\frac{v^2}{r}$$

$$-\mu_s \cancel{m} g = \cancel{m}\frac{v^2}{r}$$

$$-\mu_s g = \frac{v^2}{r}$$

$$-\mu_s g r = v^2$$

$$\pm\sqrt{-\mu_s g r} = v$$

$$v = \pm\sqrt{-0.30(-9.81m/s^2)(80.0m)}$$

$$v = \pm 15.344m/s$$

$$\boxed{v = 15m/s}$$

Universal Gravitation

Newton realized that the same gravity that keeps you on Earth also keeps the moon orbiting the Earth. This led to the idea of fields. A field is a region in space where one object can exert an influence on another object at a distance. A force is applied by one field acting on another similar field. The force of gravity involves the gravitational fields acting upon one another.

The field always points inwards towards the centre of mass. The gravitational force two objects experience are equal in magnitude but act in opposing directions. All masses have a gravitational filed, but only the gravitational fields of large masses are noticeable.

Changes in altitude and longitude affect the gravitational field strength on the surface of the Earth. Changes in the composition of the Earth's crust also affect this strength.

The gravitational field strength is the force acting on a 1 kg mass.

Let's go back to Newton and his apple…

Newton started with the idea that since the Earth, is pulling on the apple, the apple must be pulling on the Earth (Newton 3). If the apple is pulling on the Earth, that means that the object does not have to be huge to be a source of gravity. There is nothing special about the Earth compared to the apple that creates gravity. This means that any mass pulls on any other mass.

Using lots of Calculus and some impressive physics, Newton came up with two key concepts:

1) The force due to gravity between two objects is proportional to the two masses.
2) The force due to gravity is inversely proportional to the square of the distance between the two masses.

The following formula was eventually arrived at:

$\vec{F}_g = \frac{Gm_1m_2}{\vec{d}^2}$	Where Fg = force due to gravity (N) G = Universal Gravitational Constant = 6.67 x 10^{-11} Nm^2/kg^2 m1 = mass of object one (kg) m2 = mass of object two (kg) d = distance between centres of the two objects (m)

The Gravitational Constant, G, is a value that has been experimentally found. Physicists do not necessarily like formulas with constants, as it shows an incomplete understanding of a concept. Perhaps one day you will fix this embarrassing little problem for science.

Please note, G is not the same as g!

In practice, the weight of an object depends on its location with respect to one or more celestial bodies. Anything smaller doesn't exert much of a field.

Ex. You have a 200g pickles next to a 345 g sandwich on your plate. Their centres are 4.5 cm apart. Determine the force of gravity of the pickle pulling on the sandwich.

$m_1 = 200g = 0.200 kg$

$m_2 = 345g = 0.345 kg$

$d = 4.5 cm = 0.045 m$

$F_g = ?$

$\vec{F}_g = \frac{Gm_1m_2}{\vec{d}^2}$

$F_g = \frac{6.67 \times 10^{-11}(0.200kg)(0.345kg)}{(0.045m)^2}$

$\boxed{\vec{F}_g \approx 2.3 \times 10^{-9} N}$

Ex. Io is one of Jupiter's moons. It is known to have 400 active volcanoes. One of the reasons for this is because of the gravitational pull of Jupiter on Io. At some time, the Sun, Io, and Jupiter are all lined up in a straight line so that both Sun and Jupiter were pulling Io in opposite directions. Using the data below, find the net force acting on Io at this time.

Celestial Body	Distance from Io (m)	Mass (kg)
Io	-	8.93×10^{22}
Jupiter	4.22×10^{8}	1.90×10^{27}
Sun	7.98×10^{11}	1.99×10^{30}

F_S

$$F_S = \frac{G m_I m_S}{d^2}$$

$$F_S = \frac{6.67\times10^{-11}(8.93\times10^{22}\,kg)(1.99\times10^{30}\,kg)}{(7.98\times10^{11}\,m)^2}$$

$$F_S \simeq -1.86\times10^{19}\,N$$

F_J

$$F_J = \frac{G m_I m_J}{d^2}$$

$$F_J = \frac{6.67\times10^{-11}(8.93\times10^{22}\,kg)(1.90\times10^{27}\,kg)}{(4.22\times10^{8}\,m)^2}$$

$$F_J \simeq 6.35\times10^{22}\,N$$

$$F_{net} = F_S + F_J$$

$$F_{net} = -1.86\times10^{19}\,N + 6.35\times10^{22}\,N$$

$$\boxed{F_{net} \simeq 6.35\times10^{22}\,N}$$

Momentum

Momentum combines mass and velocity. It is similar to the idea of inertia – an object "has it". Inertia is different, though, because it is a concept and cannot be measured. Momentum can.

Momentum is calculated by multiplying the mass and velocity of an object.

$\vec{p} = m\vec{v}$	Where p = momentum (kgm/s) Ns m = mass (kg) v = velocity (m/s)

Ex. A 2352 kg boulder is rolling down a hill at 24 km/h. Calculate its momentum.

$m = 2352\,kg$
$\vec{v} = 24\,km/h$ [downhill]
$\hookrightarrow 6.67\,m/s$
$\vec{p} = ?$

$$\vec{p} = m\vec{v}$$

$$\vec{p} = 2352\,kg(6.67\,m/s)$$

$$\boxed{\vec{p} \simeq 16\,000\,kgm/s \text{ [downhill]}}$$

Impulse

Impulse is a change in momentum. Since implies means the momentum has changed, the objects velocity must have changed. We will assume for all of our problems that it is the velocity, and not the mass of the object, that changes. Applying this thought to our momentum equation yields:

$\Delta\vec{p} = m\Delta\vec{v} = \vec{J}$	Where Δp = change in momentum (kgm/s) m = mass of object (kg) Δv = change in velocity (m/s) J= impulse (change in momentum)

Ex. Another boulder is rolling down the hill. This boulder has a paltry mass of 23 kg and is moving at 8.2 m/s. A dog tries to stop it, but only slows it down to 2.5 m/s. What impulse did the dog apply to the boulder?

$m = 23\,kg$

$\vec{v}_1 = 8.2\,m/s$ [downhill]

$\vec{v}_2 = 2.5\,m/s$ [downhill]

$J = ?$

$\vec{J} = m\Delta\vec{v}$

$\vec{J} = 23\,kg(2.5\,m/s - 8.2\,m/s)$

$\vec{J} \approx -130\,kg\,m/s$

$\boxed{\vec{J} \approx 130\,kg\,m/s\text{ [uphill]}}$

The negative sign means the dog's impulse was acting in the negative direction. Momentum was taken away from the rock. If an impulse changes the velocity of an object, then it causes an acceleration. Where there is an acceleration, there is a force…

Newton Two Redux

When Newton first wrote his Second Law of Motion, he didn't write it as F = ma. Instead, he said that force is proportional to the rate of change in momentum:

$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$$

By substituting out formula for momentum in, one can arrive at the version of Newton's Second Law that we use. More importantly at this time, we can get:

$$\Delta\vec{p} = \vec{F}\Delta t$$

or

$$\boxed{\vec{F}\Delta t = m\Delta\vec{v}}$$

Each side of the equal sign in the equation above is equal to impulse. This is very versatile; you can use only the left side or only the right side to find the impulse, or you can use both sides to find one of the unknowns if you know the other three.

Ex. A rifle is firing a 9.00 g bullet so that it leaves the muzzle after 0.0300 s travelling at 200 m/s. Find the average force of the rifle acting on the bullet.

$m = 9.00g = 0.00900 kg$

$\Delta t = 0.0300 s$

$\vec{V}_2 = 200 m/s$

$\vec{V}_1 = 0 m/s$

$\vec{F} = ?$

$\vec{F}\Delta t = m\Delta\vec{v}$

$\vec{F} = m\frac{\Delta\vec{v}}{\Delta t}$

$\vec{F} = \frac{0.00900 kg (200 m/s - 0 m/s)}{0.0300 s}$

$\boxed{\vec{F} \approx 60 N}$

Impulse depends on two factors: force and time interval. To change an object's momentum, think of the following three situations:

Apply a medium force over a medium time interval.

$F \Delta t$

Apply a big force over a small time interval to get the same impulse.

$F_{\Delta t}$ (ouch)

Apply a small force over a long time interval and still get the same impulse.

$_F \Delta t$

Use this to explain why you would rather land on a high jump mat instead of the ground.

- the mat increases the duration of contact

Remember, it force that you feel, so you want that to be as small as possible.

Ex. After getting his Porsche out of the snowbank, 85 kg Mr. Birrell is travelling at 75 km/h when he runs into a chestnut tree.

a) If he had no airbag in his car and he came to rest against the steering wheel in 0.050s, determine how much force was exerted on his body.

$m = 85 kg$

$\Delta t = 0.050 s$

$\vec{V}_1 = 75 km/h = 20.83 m/s$

$\vec{V}_2 = 0 m/s$

$\vec{F} = ?$

$\vec{F}\Delta t = m\Delta\vec{v}$

$\vec{F} = m\frac{\Delta\vec{v}}{\Delta t}$

$\vec{F} = \frac{85 kg (0 m/s - 20.83 m/s)}{0.050 s}$

$\boxed{\vec{F} \approx -35000 N}$

b) If he did have an airbag that inflated and deflated correctly, bringing him to a rest over 0.78s, find how much force was exerted on his body.

$\Delta t = 0.78s$

$$\vec{F} = \frac{85kg\,(0m/s - 20.83m/s)}{0.78s}$$

$$\boxed{\vec{F} \simeq -2300N}$$

c) What percentage of a) is b)?

$$\frac{-2300N}{-35000N} \simeq \boxed{6.6\%}$$

Conservation of Momentum in One Dimension

Because we can measure momentum, we could measure objects before and after they have a collision. A collision between objects results in an impulse. Any collision we look at must be in an isolated system. This means that no matter or energy is allowed to enter or leave the system – there are no external forces acting on the objects.

It was noticed in Newton's day that the total momentum of all objects before a collision equals the total momentum of all objects after. This is called the Law of Conservation of Momentum and is a fundamental law of physics. Any conservation law means that whatever you started with you still have at the end.

p_a'

Before Collision	After Collision
Momentum of object a = p_a	Momentum of object a = p_a'
Momentum of object b = p_b	Momentum of object b = p_b'

The ' is pronounced "prime" and represents values after the collision.

When solving collision problems, one of two things will happen:

$p = p'$

$p_a + p_b = p_a' + p_b'$

1. The two objects will bounce apart.
2. The two objects will stick together.

Ex. Object bounce apart

ex. A 0.20kg green billiard ball moving at 9.2 m/s right hits a yellow striped ball at rest. If the green ball continues to move to the right at 2.5 m/s, find the velocity of the yellow ball.

$m_g = 0.20kg$
$v_g = 9.2m/s$
$v_y = 0m/s$
$v_g' = 2.5m/s$
$m_y = 0.20kg$
$v_y' = ?$

$\vec{p} = \vec{p}'$
$\vec{p}_g + \cancel{\vec{p}_y}^{\,0} = \vec{p}_g' + \vec{p}_y'$
$m_g\vec{v}_g = m_g\vec{v}_g' + m_y\vec{v}_y'$
$m_g v_g - m_g v_g' = m_y v_y'$
$\frac{m_g v_g - m_g v_g'}{m_y} = v_y'$

$$v_y' = \frac{0.20kg(9.2m/s) - 0.20kg(2.5m/s)}{0.20kg}$$

$$\boxed{v_y' = 6.7m/s\,[\text{right}]}$$

Ex. Objects stick together

A 95 kg linebacker tackles a 75 kg running back. If the linebacker was moving towards the south goalpost at 8.4 m/s while the running back is moving at 5.3 m/s North, determine their final velocity.

N ↑

$m_L = 95\,kg$

$m_r = 75\,kg$

$V_L = -8.4\,m/s$

$V_R = 5.3\,m/s$

$V_{LR} = ?$

$\vec{p} = \vec{p}\,'$

$\vec{P}_L + \vec{P}_R = \vec{P}_{LR}$

$m_L \vec{V}_L + m_R \vec{V}_R = (m_L + m_R)\vec{V}_{LR}$

$\dfrac{m_L \vec{V}_L + m_R \vec{V}_R}{(m_L + m_R)} = \vec{V}_{LR}$

$\vec{V}_{LR} = \dfrac{95(-8.4\,m/s) + 75(5.3)}{(95\,kg + 75\,kg)}$

$\vec{V}_{LR} = -2.356\,m/s$

$\vec{V}_{LR} \approx -2.4\,m/s$

$\boxed{\vec{V}_{LR} \approx 2.4\,m/s\,[\text{south}]}$

While the above example showed two objects sticking together after the collision, it is also possible to have the reverse. Sometimes, the objects start together and end up apart after the collision. If this happens, just reverse the right and left sides of the equation in the above example.